Computing Word Senses by Semantic Mirroring and Spectral Graph Partitioning

Masters Thesis by Martin Fagerlund

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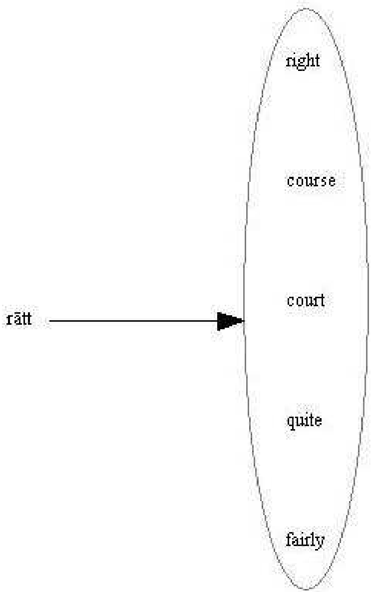
Outline

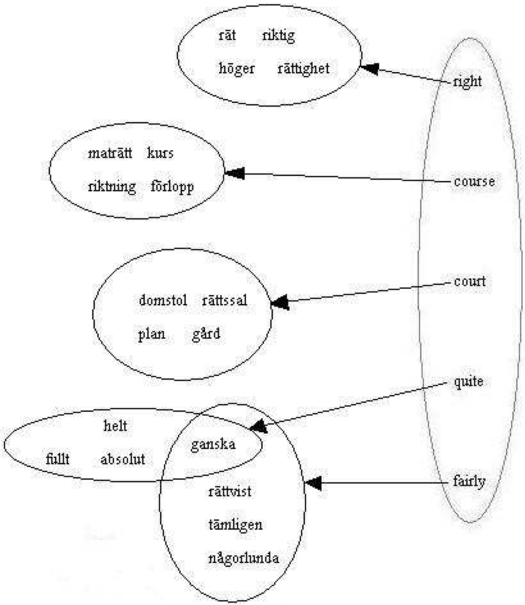
Semantic Mirrors (Dyvik)

- Use translations to extract information from a bilingual lexicon.
- Semantically closely related words tend to have strongly overlapping sets of translations.
- Words with a wide meaning tend to have a higher number of translations, than words with a more narrow meaning.

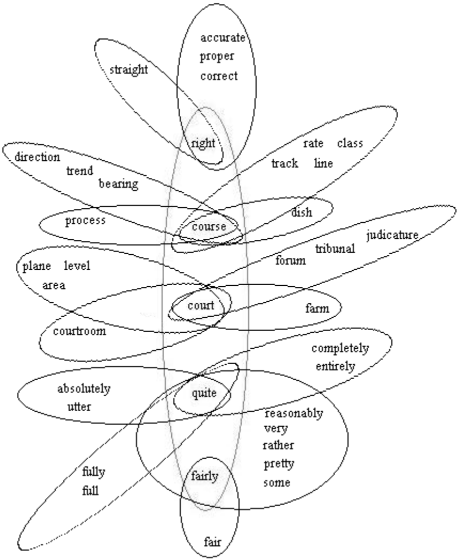
Semantic Mirrors



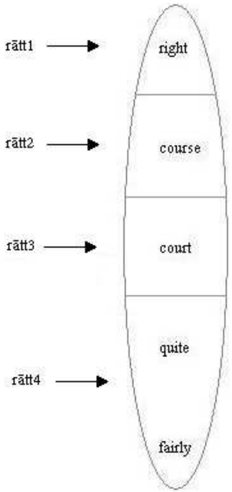
Semantic Mirrors



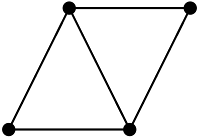
Semantic Mirrors



Semantic Mirrors

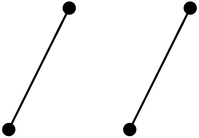


Graph theory



- Connected graph with vertex set V and edge set E.

Graph theory



- Disconnected graph.

Graph theory

- The weight function.
- $w(v_i, v_j) = w(v_j, v_i)$.
- $w(v_i, v_j) \geq 0$.

- The adjacency matrix.

$$A(i,j) = \begin{cases} w(v_i, v_j), & \text{if } v_i \text{ and } v_j \text{ are adjacent,} \\ 0, & \text{otherwise.} \end{cases}$$

Graph theory - Graphs and Matrices

- The Laplacian.

$$d(v_i) = \sum_j w(v_i, v_j).$$

$$D(i,j) = \begin{cases} d(v_i), & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$$

- $L = D - A$.

Graph theory - Graphs and Matrices

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Graph theory - Graphs and Matrices

- The normalised Laplacian.

$$\mathcal{L} = D^{-1/2} L D^{-1/2}.$$

$$\mathcal{L} = \begin{cases} 1 - \frac{w(v_i,v_i)}{d(v_i)}, & \text{if } i = j \text{ and } d(v_i) \neq 0, \\ -\frac{w(v_i,v_j)}{\sqrt{d(v_i)d(v_j)}}, & \text{if } v_i \text{ and } v_j \text{ are adjacent,} \\ 0, & \text{otherwise.} \end{cases}$$

Spectral graph partitioning

Compute the eigenvalues

$$0 = \lambda_0 \leq \lambda_1 \leq \ldots \leq \lambda_{n-1},$$

and the eigenvectors

$$\bar{u}_0, \ldots, \bar{u}_{n-1}$$

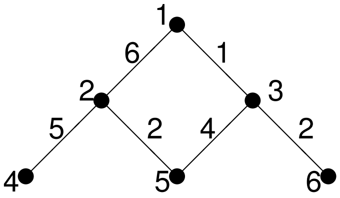
of $\mathcal{L}$.
λ_1 is called the Fiedler value, and $\bar{u}_1$ is called the Fiedler vector.

Spectral graph partitioning

Let $\bar{u} = (u_1, \ldots, u_n)$ be the Fiedler vector.
Divide the vertices into S and $\bar{S}$ using a splitting value s:
$v_i \in S$ if $u_i > s$
$v_i \in \bar{S}$ if $u_i \leq s$.
A few ways of choosing s:

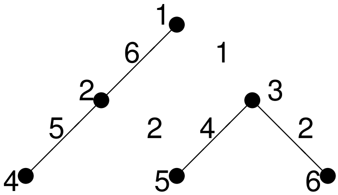
- bisection: s is the median of $\{u_1, \ldots, u_n\}$.
- sign cut: s =0.
- gap cut: s is a value in the largest gap in the sorted Fiedler vector.

Spectral graph partitioning - An example



$$\begin{pmatrix} -0.2523 \\ -0.4183 \\ 0.5701 \\ -0.3603 \\ 0.3571 \\ 0.4232 \end{pmatrix}$$

Spectral graph partitioning - An example



$$\begin{pmatrix} -0.2523 \\ -0.4183 \\ 0.5701 \\ -0.3603 \\ 0.3571 \\ 0.4232 \end{pmatrix}$$

Spectral graph partitioning

Measures of the partitioning. Let

$$d(S) = \sum_{v_i \in S} d(v_i),$$

and

$$|\,E(S, \bar{S})\,| = \sum_{v_i \in S, v_j \in \bar{S}} w(v_i, v_j).$$

Conductance,

$$\phi(S) = d(V)\frac{|\,E(S, \bar{S})\,|}{d(S)d(\bar{S})},$$

Spectral graph partitioning

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and

$$|E(S, \bar{S})| = \sum_{v_i \in S, v_j \in \bar{S}} w(v_i, v_j).$$

Sparsity,

$$sp(S) = \frac{|E(S, \bar{S})|}{\min(d(S), d(\bar{S}))}$$

Spectral graph partitioning

The conductance of a graph

$$\phi_G = \min_S \phi(S)$$

The sparsity of a graph

$$sp_G = \min_S sp(S)$$

Spectral graph partitioning

The Cheeger inequalities

$$2\phi_G > \lambda_1 \geq \frac{\phi_G^2}{8}.$$

$$2sp_G \geq \lambda_1 \geq \frac{sp_G^2}{2}.$$

Separating word senses - Translation

- English-Swedish lexicon of adjectives.
  Words into two lists.
- Translation matrix **B**.
  Rows correspond to English words.
  Columns correspond to Swedish words.

Separating word senses - Translation

- English word j defines a vector $\bar{e}_j$ by

$$\bar{e}_j(i) = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases}$$

- $\mathbf{B}^T \bar{e}_j$ gives translations from English to Swedish.

Separating word senses - Computation

Start with an English word, called *eng1*.

$$\mathbf{B}\mathbf{B}^T \bar{e}_{eng1}.$$

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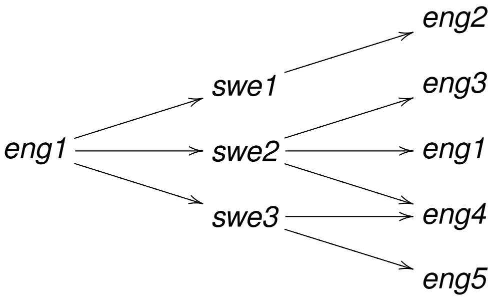

Separating word senses - Computation

Replace the row in **B** corresponding to *eng*1, with an all-zero row. Call this new matrix $\mathbf{B}_{mod1}$.

$$\mathbf{B}_{mod1}\mathbf{B}^T\bar{e}_{eng1}.$$

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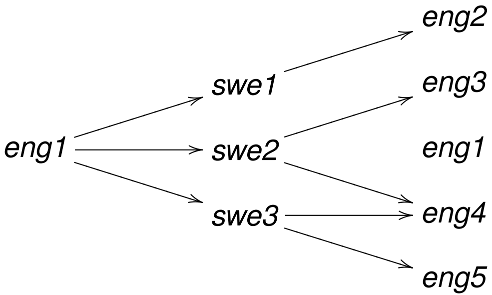
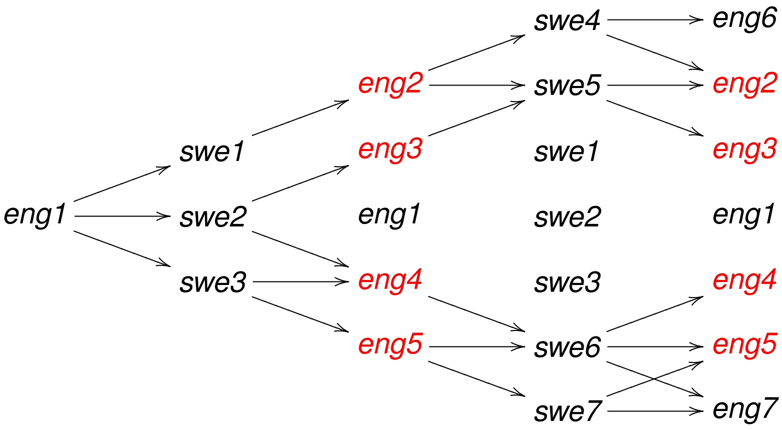


Separating word senses - Computation

Let $\mathbf{B}_{mod2}$ be the matrix **B**, with columns corresponding to the Swedish words *swe1,...,swe3*, replaced with all-zero columns.
Then

$$\mathbf{B}_{mod2}\mathbf{B}_{mod2}^{T}\mathbf{E}$$

Separating word senses - Computation



Separating word senses - Computation

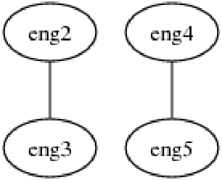
If in

$$\mathbf{B}_{mod2}\mathbf{B}_{mod2}^{T}\mathbf{E}$$

keeping only the rows corresponding to *eng2,...eng5*, we get a symmetric matrix **A.**

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

Separating word senses - Computation



Separating word senses - Computation

- Create $\mathcal{L}$, and compute the Fiedler vector.
- Sort the Fiedler vector, and make $n - 1$ cuts.
- For each cut, compute the conductance and choose the the cut that produce the lowest value.
- Partition the graph.

Separating word senses - Computation

- Continue and partition the largest of the subgraphs obtained.
- End the iteration when a stopping criterion is fulfilled.

Separating word senses - Example 1

Global

absolute	full-scale	round
aggregate	intact	teetotal
all-out	integral	total
clear	integrate	unbroken
complete	international	universal
entire	mondial	utter
full	one-piece	whole
full-length	outright	worldwide

Separating word senses - Example 1

Global

intact whole full-length one-piece clear round full	absolute utter complete all-out teetotal outright entire total	integrate unbroken integral	aggregate full-scale
worldwide international	mondial	universal	

Separating word senses - Example 2

Visiting

adventitious
alien
exotic
foreign
outlandish
strange
unaccustomed
uncouth
unfamiliar
ungenial

Separating word senses - Example 2

Visiting

alien foreign outlandish exotic	unaccustomed unfamiliar strange
adventitious	uncouth
ungenial	

Evaluation and Results

Evaluation

- 10 random words with at least 8 vertices in the graph were evaluated.
- Both conductance and sparsity were used as a measure.

Results

- The sense groups are not always completely synonymous with the seed word.
- The words are often consistent within the sense groups.

Evaluation and Results

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Conclusions and future work

Conclusion

Our preliminary results indicate that graph partitioning-based semantic mirroring can be developed into an automatic method for computing word senses.

Future work

- automatic treatment of homonyms
- Tests on other dictionaries, languages
- Systematic evaluation of the method

To get a copy of Martin Fagerlund's Masters thesis, send me or him a message.